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Original Article

Quasi-Static transient Thermal Stresses in a Robin's thin Rectangular plate with internal moving heat source

D. T. Solanke and M. H. Durge

D. T. Solanke and M. H. Durge

From
Sudhakar Naik and Umashankar Khetan college Akola, Maharashtra State, India.
Anand Niketan Collge, Warora, Maharashtra State, India.

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ABSTRACT

This paper concern with the transient non-homogeneous thermoelastic problem with Robin's boundary condition in thin rectangular plate of isotropic material, occupying region R: $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$ where $b < a, c < b$, c is very small as compared to b. Initial temperature of the plate is $f(x, y, z)$ and plate is placed in an ambient temperature zero. The plate is subjected to the activity of moving point heat source at point (x', y', z') moving with constant velocity vector $\bar{u} = u_1 i + u_2 j + u_3 k$ where u_1, u_2, u_3 are the component of velocity vector along x, y, z axes respectively. The heat conduction equation containing heat generation term is solved by applying integral transform technique and Green's theorem is adopted in deriving the solution of heat conduction equation. The solution is obtained in a series form of trigonometric function and the thermal stresses are derived.

Keywords:

Robin's thin rectangular plate, moving heat source, thermal stresses, Green's theorem.

2.Introduction:

During the second half of 20th century, non-isothermal problems of the theory of elasticity became increasingly important. This is due to their wide application in diverse fields. The high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses that reduce the strength of aircraft structure. Recently D. T. solanke, S. M. Durge have studied the thermal stresses, in thin solid cylinder and hollow cylinder with Dirichlet's, Neumann's and Robin's boundary condition and rectangular plates from [1] to [6]. Nobody previously have studied such type of three dimensional temperature distribution and thermal stresses with moving heat source in thin rectangular plate with Robin's boundary condition. This is new contribution to the field of thermoelasticity. In this present paper we determine temperature distribution and thermal stresses in thin rectangular plate with moving point heat source with Robin's type boundary condition. The heat conduction equation containing heat generation term is solved by applying integral transform technique and Green's theorem is adopted in deriving the solution of heat conduction equation. The solution is obtained in a series form of trigonometric function and the thermal stresses are derived.

3.Formulation of the problem:

Consider a thin rectangular plate of isotropic material of length a, breadth b and height c; occupying the region R: $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$ where $b < a, c < b$, c is very small as compared to b. Initial temperature of the plate is $f(x, y, z)$ placed in an ambient temperature zero. The plate is subjected to the activity of instantaneous moving point heat source at the point (x', y', z') which changes its place along x, y, z axes moving with constant velocity vector $\bar{u} = u_1 i + u_2 j + u_3 k$ where u_1, u_2, u_3 are component of velocity vector along x, y, z axes respectively. The activity of moving heat source or initial temperature of the plate may cause the generation of heat due to nuclear interaction that may be a function of position and time in the form $g(x, y, z, t)$ w/s^3. The temperature distribution of the rectangular plate is described by the differential equation of heat conduction with heat generation term as in [7] page no. 9 is given by

$$\nabla^2 T + \frac{1}{k} g = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where $T = T(x, y, z)$ is temperature distribution, k is thermal conductivity of the material of the plate, $\alpha = \dfrac{k}{\rho c_p}$ is thermal diffusivity, ρ is density, c_p is specific heat of the material and g is volumetric energy(heat) generation term. ∇^2 is Laplacian operator in rectangular coordinates in three dimension. Now consider an instantaneous moving point heat source at a point (x', y', z') and releasing its heat spontaneously at time τ. Such volumetric moving heat source in rectangular coordinates is given by $g(x, y, z, t) = g_p{}^i \delta(x - x')\delta(y - y')\delta(z - z')\delta(t - \tau)$

Hence above equation reduces to

$$\nabla^2 T + \frac{1}{k} g_p{}^i \delta(x - x')\delta(y - y')\delta(z - z')\delta(t - \tau) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{3.1}$$

Where $x' = u_1 t$, $y' = u_2 t$, $z' = u_3 t$ (3.2)

With initial and boundary condition

$$\frac{\partial T}{\partial x} - hT = 0 \quad \text{at} \quad x = 0 \tag{3.3}$$

$$\frac{\partial T}{\partial x} + hT = 0 \quad \text{at} \quad x = a \tag{3.4}$$

$$\frac{\partial T}{\partial y} - hT = 0 \quad \text{at} \quad y = 0 \tag{3.5}$$

$$\frac{\partial T}{\partial y} + hT = 0 \quad \text{at} \quad y = a \tag{3.6}$$

$$\frac{\partial T}{\partial z} - hT = 0 \quad \text{at} \quad z = 0 \tag{3.7}$$

$$\frac{\partial T}{\partial z} + hT = 0 \quad \text{at} \quad z = a \tag{3.8}$$

$$T = f(x, y, z) \quad \text{at} \quad t = 0, \tau = -\infty \tag{3.9}$$

4. Formulation of the thermoelastic problem:

Let us introduce a thermal stress function χ related to component of stress in the rectangular coordinates system as in [8] where $\chi = \chi_c + \chi_p$, χ_c is complementary solution and χ_p is particular solution. χ_c and χ_p are governed by a linear homogeneous differential equation and linear non-homogeneous differential equation

$$\nabla^4 \chi_c = 0 \tag{4.1}$$

$$\nabla^2 \chi_p = -\alpha E \Gamma \tag{4.2}$$

Where Γ is temperature change $\Gamma = T - T_i$ where T_i is initial temperature

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ since plate is thin z is negligible

$$\sigma_{xx} = \frac{\partial^2 \chi}{\partial y^2} \tag{4.3}$$

$$\sigma_{yy} = \frac{\partial^2 \chi}{\partial x^2} \tag{4.4}$$

$$\sigma_{xy} = -\frac{\partial^2 \chi}{\partial x \partial y} \tag{4.5}$$

The boundary condition is $\sigma_{xy} = \sigma_{yy} = 0$ at $y = b$ (4.6)

5.Solution:

We define integral transform of $T(x, y, z, t)$ by

$$\overline{T}(\alpha_m, \beta_m, \gamma_m, t) = \int_R T(x, y, z, t) X(\alpha_m x) Y(\beta_m y) Z(\gamma_m z) dv \tag{5.1}$$

And its inverse integral transform by

$$T(x, y, z, t) = \sum_{m=0}^{\infty} \frac{\overline{T}(\alpha_m, \beta_m, \gamma_m, t) X(\alpha_m x) Y(\beta_m y) Z(\gamma_m z)}{\eta_m} \tag{5.2}$$

Where
$$X(\alpha_m x) = \alpha_m \cos(\alpha_m x) + H \sin(\alpha_m x) \tag{5.3}$$

$$Y(\beta_m y) = \beta_m \cos(\beta_m y) + H \sin(\beta_m y) \tag{5.4}$$

$$Z(\gamma_m z) = \gamma_m \cos(\gamma_m z) + H \sin(\gamma_m z) \tag{5.5}$$

$$H = \frac{h}{k} \tag{5.6}$$

η_m is product of the normalization integral and

$$\eta_m = N(\alpha_m) N(\beta_m) N(\gamma_m)) = \left[\left(\alpha_m^2 + H^2 \right) \tfrac{a}{2} + H \right] \left[\left(\beta_m^2 + H^2 \right) \tfrac{b}{2} + H \right] \left[\left(\gamma_m^2 + H^2 \right) \tfrac{c}{2} + H \right] \tag{5.7}$$

α_m are roots of the transcendental equation $\tan(\alpha_m a) = \frac{2\alpha_m H}{\alpha_m^2 - H^2}$ (5.8)

β_m are roots of the transcendental equation $\tan(\beta_m b) = \frac{2\beta_m H}{\beta_m^2 - H^2}$ (5.9)

γ_m are roots of the transcendental equation $\tan(\gamma_m c) = \frac{2\gamma_m H}{\gamma_m^2 - H^2}$ (5.10)

Taking integral transform of equation (3.1) and using boundary condition and following Green's theorem we obtain

$$\int_R \nabla^2 T \psi_k dv = \int_R T \nabla^2 \psi_k dv + \sum_{i=1}^{N} \int_{s_i} \left[\psi_k \frac{\partial T}{\partial n_i} - T \frac{\partial \psi_k}{\partial n_i} \right] ds_i \tag{5.11}$$

$$\frac{d\overline{T}}{dt}+\alpha(\alpha_m{}^2+\beta_m{}^2+\gamma_m{}^2)\overline{T}=\frac{\alpha}{k}g_p{}^iX(\alpha_m x')Y(\beta_m y')Z(\gamma_m z')\delta(t-\tau)$$

$$\overline{T}=\left[\overline{f}(\alpha_m,\beta_m,\gamma_m)+\frac{\alpha}{k}g_p{}^iX(\alpha_m x')Y(\beta_m y')Z(\gamma_m z')e^{\alpha(\alpha_m{}^2+\beta_m{}^2+\gamma_m{}^2)\tau}\right]e^{-\alpha(\alpha_m{}^2+\beta_m{}^2+\gamma_m{}^2)t}$$

Taking inverse integral transform we obtain

$$T=\sum_{m=0}^{\infty}\frac{X(\alpha_m x)Y(\beta_m y)Z(\gamma_m z)}{\eta_m}\psi\, e^{-\alpha(\alpha_m{}^2+\beta_m{}^2+\gamma_m{}^2)t} \tag{5.12}$$

Where $\psi=\left[\overline{f}(\alpha_m,\beta_m,\gamma_m)+\frac{\alpha}{k}g_p{}^iX(\alpha_m x')Y(\beta_m y')Z(\gamma_m z')e^{\alpha(\alpha_m{}^2+\beta_m{}^2+\gamma_m{}^2)\tau}\right]$ (5.13)

$$\Gamma=\sum_{m=0}^{\infty}\frac{X(\alpha_m x)Y(\beta_m y)Z(\gamma_m z)}{\eta_m}\psi\, f(t) \tag{5.14}$$

Where $f(t)=\left[e^{-\alpha(\alpha_m{}^2+\beta_m{}^2+\gamma_m{}^2)t}-1\right]$ (5.15)

6.Solution of Thermoelastic problem:

Let the suitable form of χ_c satisfying (4.1) be

$$\chi_c=\sum_{m=0}^{\infty}y\left[Ae^{\alpha_m y}+Be^{-\alpha_m y}\right]\cos(\alpha_m x)+y\left[Ce^{\alpha_m y}+De^{-\alpha_m y}\right]\sin(\alpha_m x) \tag{6.1}$$

Let the suitable form of χ_p satisfying (4.2) be

$$\chi_p=\alpha E\sum_{m=}^{\infty}\frac{X(\alpha_m x)Y(\beta_m y)Z(\gamma_m z)}{(\alpha_m{}^2+\beta_m{}^2)\eta_m}\psi\, f(t) \tag{6.2}$$

$$\chi=\sum_{m=0}^{\infty}y\left[Ae^{\alpha_m y}+Be^{-\alpha_m y}\right]\cos(\alpha_m x)+y\left[Ce^{\alpha_m y}+De^{-\alpha_m y}\right]\sin(\alpha_m x)+\alpha E\frac{X(\alpha_m x)Y(\beta_m y)Z(\gamma_m z)}{(\alpha_m{}^2+\beta_m{}^2)\eta_m}\psi\, f(t) \tag{6.3}$$

From (4.3) and (6.3) we obtain

$$\sigma_{xx}=\sum_{m=0}^{\infty}\left[y\left(A\alpha_m{}^2e^{\alpha_m y}+B\alpha_m{}^2e^{-\alpha_m y}\right)+2\left(A\alpha_m e^{\alpha_m y}-B\alpha_m e^{-\alpha_m y}\right)\right]\cos\alpha_m x+\left[y\left(C\alpha_m{}^2e^{\alpha_m y}+D\alpha_m{}^2e^{-\alpha_m y}\right)+2\left(C\alpha_m e^{\alpha_m y}-D\alpha_m e^{-\alpha_m y}\right)\right]\sin\alpha_m x-$$

$$\frac{\alpha E\beta_m{}^2X(\alpha_m x)Y(\beta_m y)Z(\gamma_m z)}{(\alpha_m{}^2+\beta_m{}^2)\eta_m}\psi\, f(t) \tag{6.4}$$

From (4.4)and (6.3) we obtain

$$\sigma_{yy}=\sum_{m=0}^{\infty}y\left(Ae^{\alpha_m y}+Be^{-\alpha_m y}\right)\left(-\alpha_m{}^2\cos(\alpha_m x)\right)+y\left(Ce^{\alpha_m y}+De^{-\alpha_m y}\right)\left(-\alpha_m{}^2\sin(\alpha_m x)\right)-\frac{\alpha E\alpha_m{}^2X(\alpha_m x)Y(\beta_m y)Z(\gamma_m z)}{(\alpha_m{}^2+\beta_m{}^2)\eta_m}\psi\, f(t) \tag{6.5}$$

From (4.5) and (6.3) we obtain

$$\sigma_{xy}=\sum_{m=0}^{\infty}\left[y\left(A\alpha_m{}^2e^{\alpha_m y}-B\alpha_m{}^2e^{-\alpha_m y}\right)+\left(A\alpha_m e^{\alpha_m y}+B\alpha_m e^{-\alpha_m y}\right)\right]\sin\alpha_m x-\left[y\left(C\alpha_m{}^2e^{\alpha_m y}-D\alpha_m{}^2e^{-\alpha_m y}\right)+\left(C\alpha_m e^{\alpha_m y}+D\alpha_m e^{-\alpha_m y}\right)\right]\cos\alpha_m x-$$

$$\frac{\alpha E[-\alpha_m{}^2\sin(\alpha_m x)+H\alpha_m\cos(\alpha_m)]Y'(\beta_m y)Z(\gamma_m z)}{(\alpha_m{}^2+\beta_m{}^2)\eta_m}\psi\, f(t) \tag{6.6}$$

Applying condition (4.6) to (6.5) and (6.6) we obtain

$$A = \frac{\alpha E Y(\beta_m b) Z(\gamma_m z) e^{-\alpha_m b}}{2b^2(\alpha_m^2 + \beta_m^2)\eta_m} \psi f(t)\left[1 - b(\alpha_m - H)\right] \tag{6.7}$$

$$B = -\frac{\alpha E Y(\beta_m b) Z(\gamma_m z) e^{\alpha_m b}}{2b^2(\alpha_m^2 + \beta_m^2)\eta_m} \psi f(t)\left[1 + b(\alpha_m + H)\right] \tag{6.8}$$

$$C = \frac{\alpha E H Y(\beta_m b) Z(\gamma_m z) e^{-\alpha_m b}}{2b^2\alpha_m(\alpha_m^2 + \beta_m^2)\eta_m} \psi f(t)\left[1 - b(\alpha_m - H)\right] \tag{6.9}$$

$$D = -\frac{\alpha E H Y(\beta_m b) Z(\gamma_m z) e^{\alpha_m b}}{2b^2\alpha_m(\alpha_m^2 + \beta_m^2)\eta_m} \psi f(t)\left[1 + b(\alpha_m + H)\right] \tag{6.10}$$

Substituting these value in above equations we obtain

$$\sigma_{xx} = \sum_{m=0}^{\infty} \frac{\alpha E Y(\beta_m b) Z(\gamma_m z)}{b^2(\alpha_m^2 + \beta_m^2)\eta_m} \psi f(t)$$

$$\left\{ \begin{array}{l} \left[\alpha_m^2 y\left(\sinh\alpha_m(y-b) - b\alpha_m\cosh\alpha_m(y-b) + bH\sinh\alpha_m(y-b)\right) + 2\alpha_m\left(\cosh\alpha_m(y-b) - b\alpha_m\sinh\alpha_m(y-b) + bH\cosh\alpha_m(y-b)\right)\right]\cos\alpha_m x + \\ \left[\alpha_m y\left(H\sinh\alpha_m(y-b) - b\alpha_m H\cosh\alpha_m(y-b) + bH^2\sinh\alpha_m(y-b)\right) + 2\left(H\cosh\alpha_m(y-b) - b\alpha_m H\sinh\alpha_m(y-b) + bH^2\cosh\alpha_m(y-b)\right)\right]\sin\alpha_m x \end{array} \right\} -$$

$$\frac{\alpha E \beta_m^2 X(\alpha_m x) Y(\beta_m y) Z(\gamma_m z)}{(\alpha_m^2 + \beta_m^2)\eta_m} \psi f(t) \tag{6.11}$$

$$\sigma_{yy} = \sum_{m=0}^{\infty} \frac{\alpha E Z(\gamma_m z)}{(\alpha_m^2 + \beta_m^2)\eta_m} \psi f(t) \left\{ \begin{array}{l} \dfrac{Y(\beta_m b)}{b^2} \left\langle \begin{array}{l} \left[b\alpha_m\cosh\alpha_m(y-b) - \sinh\alpha_m(y-b) - bH\sinh\alpha_m(y-b)\right]\left(y\alpha_m^2\cos(\alpha_m x)\right) + \\ \left[b\alpha_m H\cosh\alpha_m(y-b) - H\sinh\alpha_m(y-b) - bH^2\sinh\alpha_m(y-b)\right]\left(y\alpha_m\sin(\alpha_m x)\right) \end{array} \right\rangle - \\ \alpha_m^2 X(\alpha_m x) Y(\beta_m y) \end{array} \right\} \tag{6.12}$$

$$\sigma_{xy} = \frac{\alpha E Y(\beta_m b) Z(\gamma_m z)}{b^2(\alpha_m^2 + \beta_m^2)\eta_m} \psi f(t)$$

$$\left\{ \begin{array}{l} \left[\alpha_m^2 y\left(\cosh\alpha_m(y-b) - b\alpha_m\sinh\alpha_m(y-b) + bH\cosh\alpha_m(y-b)\right) + \alpha_m\left(\sinh\alpha_m(y-b) - b\alpha_m\cosh\alpha_m(y-b) + bH\sinh\alpha_m(y-b)\right)\right]\sin\alpha_m x + \\ \left[\alpha_m y\left(b\alpha_m H\sinh\alpha_m(y-b) - H\cosh\alpha_m(y-b) - bH^2\cosh\alpha_m(y-b)\right) + \left(b\alpha_m H\cosh\alpha_m(y-b) + bH^2\sinh\alpha_m(y-b) - H\sinh\alpha_m(y-b)\right)\right]\cos\alpha_m x \end{array} \right\}$$

$$-\frac{\alpha E X'(\alpha_m x) Y'(\beta_m y) Z(\gamma_m z)}{(\alpha_m^2 + \beta_m^2)\eta_m} \psi f(t) \tag{6.13}$$

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